Tangent line worksheet calculus

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Secondly, the issue of change that we will be considering is one of the most important concepts we will face in the second chapter of this course. In fact, this is probably one of the most important concepts that we will encounter throughout the course. So looking at it now will help us start thinking about it from the beginning. Tangent Lines The first problem we're going to look at is the tangent line problem. Before you get into this problem, it would probably be better to define a tangent line. The touchline to the function (f(x) at point (x and a) is a line that simply touches the function graph at the point in question and is a parallel (in some way) graph at that point. Take a look at the graph below. In this graph, the line is a tangent line at a given point because it simply touches the graph at that point, as well as parallel graphics at that point. Similarly, at the second point shown, the line is just to touch the chart at this point, but it's not parallel to the graph at that point, and so it's not a touchline to the graph at that point. In the second point shown (the point where the line is not a tangent line) we sometimes call the line the secant line. We used the word in parallel a couple of times and now we probably have to be a little careful with it. In general, we will think of the line and the graph as parallel at the point if they are both moving in the same direction at that point. So, at the first point above the graph and the line is moving in the same direction, and so we'll say they're parallel at that point. At the second point, on the other hand, the line and graph do not move in the same direction, so they are not parallel at this point. Okay, now that we've got a tangent line out of the way, let's move on to the tangent line problem. This is probably best done with an example. Example 1 Find a tangent line on the th (left (x (right) 15 - 2 x2 in (x'1). Show solution We know from algebra that to find the equation line we need either two points on the line or dots on the line and the line tilt. Since we know that we are after the tangent line we have a point that is on the line. On the tangent line and the schedule of function should be touched at No(x) No. 1, so the point (left (1,f e left) right) left) left (1,13)) should be on the line. to allow us to find a slope tangent line let's just focus on if we can identify the slope of the tangent line. At this point all we're going to be able to do is get an estimate for the slope tangent line, but if we do it right, we should be able to get an estimate that's actually a touchline slope. We'll do this by starting with the point that we're after, let's call it (P) (Then we'll select another point that lies on the function graph, let's call this point (left) (x,f'left (x (right)). For the argument let's choose (x 2), and so the second point will be (left). that the lines secant and tangent are somewhat similar, and so the incline of the secant line should be somewhat close to the actual incline of the tangent line. So, as an assessment of the incline, we can use the tilt of the sequest line, Let's call it (m_) who, (m_) - (frak) left (2) - right (1 (right) - 1 (right) - 2 - 1 frak 7 - 13 {1} - 6 Now, if we weren't too interested in accuracy, we could say it's good enough, and use that as an assessment of the ablet of the tangent. We would appreciate that at least somewhat closely the actual value. So, to get a more complete estimate, we can take (x) that closer to (x 1) and rework the work above to get a new assessment on the slope. Then we could take the third value (x) even closer and get an even better score. In other words, as we get closer and closer to the sequencing line connecting (yap) and (P) to get closer and closer to the incline of the tangent line, we will approach and approach the incline of the tangent line. If you're browsing this online, the picture below shows the process. As you can see (if you're reading this online) as we moved in closer and closer to the (P) line the secant starts to look more and more like a touchline and so the approximate slopes (i.e. the slopes of the secant lines) are getting closer and closer to the exact slope. Also, don't worry about how I got the exact or approximate slopes. We will be calculating the approximate slopes soon, and we will be able to calculate the exact slope in several sections. On this figure, we only on what that to the right of K (PL), but we could just as easily use q (I) that were to the left of (P) and we would get the same results. In fact, we should always take a look at (me) in that on both sides of (P). In this case, the same thing happens on both sides of the (PL). However, we eventually see that should not happen. Therefore, we should always look at what is happening on both sides of view in the implementation of this kind of process. So let's see if we can come up with the approximate slopes that we showed above, and therefore estimate the slope of the tangent line. In order to simplify this process, let's take a formula for tilting the line between (P) and (I) (m_) that will work for anyone (x) with which we choose to work. (m_) - frak left (x right) - fae left (1 right) - 1 - frak 15 - 2 x2 - 13 x - 1 Frak 2 - 2 x2 x - 1 Now, let's choose some values (x) closer and closer to (x 1), connect and get some slopes. (x) (m_) (x)) ((m_) 2 -6 0 -2 1.5 -5 0.5 -3 1.1 -4.2 0.9 -3.8 1.01 -4.02 0.99 -99 3.98 1.001 -4.002 0.999 -3.998 1.0001 -4.0002 0.9999 -3.9998 So if we take (x) to the right of 1 and move them very close to 1, it seems that the tilt of the secant lines, it seems to be approaching -4. Similarly, if we take (x) to the left of 1 and move them very close to 1 tilt lines secant again appears to be approaching -4. Based on this data it seems that the slopes of the secant lines are approaching -4 as we move in the direction (x 1), so we will evaluate that the slope of the tangent line is also -4. As noted above, this is the right value and we will be able to prove it eventually. Now, the line equation that runs through the left (a,f'left (right) is given y f on the left (right) equation of the tangent line to (f'left (x 'right) - 15 - 2 'x'2') in No (x'1) - it's -y 13 - 4 left (right) - 4x - 17 There are a few important moments, To mark our work above. that what happens on one side of the point will also happen on the other side. We should always look at what is happening on both sides of the point. In this example we could sketch out a graph and from this assumption that what happens on the one hand will also happen on the other side, but we usually don't have the graphics in front of us or be able to easily get them. Next, note that when we say that we are going to move in close proximity to the point in question we mean that we are going to move in very close and we also used more than just a couple of points. We should never try identify a trend based on multiple points that aren't really all that close to the point in question. The next thing to note is actually a warning more than anything. The values in this example were m_ pretty good and it was pretty clear what value they were approaching after a few calculations. In most cases, this is not the case. Most values will be much dirtier, and you often need quite a few calculations to be able to get a score. You should always use at least four points on each side to get a score. Two points is never enough to get a good score and three points also often won't be enough to get a good score. Typically, you keep collecting points closer and closer to the point you look at until the change in value between the two consecutive points becomes very small. Finally, we were after what was going on on th (x 1) and we couldn't actually plug (x 1) into our formula for the slope. Despite this limitation, we were able to identify some information about what was going on on th (x 1) just by looking at what was going on around (x 1). This is more important than you might first understand, and we will discuss this point in detail in later sections. Before we move on let's do a quick review of just what we did in the example above. We wanted the touchline to be at the point (x x x) at the point (x th a). First, we know that the dot (P) (P) (left, left (right)) will be on the tangent line. Next, we'll take the second point, which is on the function graph, Let's call it ((left, left (x (right)) and calculate the tilt of the line connecting (P) and (I) as follows, (m_ (AP) - Frak on the left (x (right) - f'left (right) - a' We then take the values (x) that are closer and closer to (x) (making sure that look at q (x) on both sides of the list Then the tangent line will be Y (left) and left (x - a) The speed of change The next problem we need to look at is the speed at which the problem changes. As mentioned earlier, this will be one of the most important concepts that we will consider throughout this course. Here we look at a function (left (x on the right) that represents a certain amount that varies depending on how the car changes (x). but it makes examples that are easy to visualize. What we want to do here is determine how quickly (on the left (x (right) changes at some point, say (x x h a). changes, and sometimes just the rate of change change (x x th a). As with the tangent line problem, all we're going to be able to do at this point is to assess the rate of change. So let's go ahead with the examples above and think about what (left (right) is like something that changes over time, and that measurement time. what we need to do is choose another point, say (x), and then the average change rate will be,start the alignment .A.R.C. - frak-mboxchanges in f on the left (x right) - f left (right) instant change rate in (x) All we have to do is select the values (x) closer and closer to (x y a) (be sure to pick them on either side of q (x)) and calculate the values (A.R.C.) We can estimate the instantaneous rate of change from this. that the amount of air in the balloon after an hour (t)) is given to the V on the left (t (right) - 6 t 2 and 35 Estimate the instantaneous rate of volume change in 5 hours. Show the solution well. (frak) on the left (t right) - We-left (right) To estimate the instantaneous rate of volume change at 25 t - 5 euros to estimate the instantaneous rate of volume change at 5 euros, we just need to select values that are getting closer and closer to z (t 5). Here is the table of values (t) and the average rate of change for these values. (etc.) (A.R.C.) (t) (A.R.C.) 6 25.0 4 7.0 5.5 19.75 4.4. 5 10.75 5.1 15.91 4.9 14.11 5.01 15,0901 4.99 14.9101 5.001 15.0009001 4.9999 14.991001 5.0001 15.00090001 4.99999 14.99910001 Thus, of this table seems, that the average rate of change is approaching 15, and so we can estimate that the instantaneous rate of change is 15 at the moment. So the only thing that this tells us about the volume on Let's put some units on the answer from above. This can help us see what's going on with the volume at the moment. Let's say the units by volume were in cm3. Units by change rate (both medium and instant) then cm3/hr. We calculated that at 15 cm3/h the volume varies at 15 cm3/h. This means that the volume changes in such a way that if the speed was constant, in an hour the balloon would have 15 cm3 more air than it was at the t level (t 5). We have to be careful here though. In fact, there probably won't be 15 cm3 more air in the balloon in an hour. The speed at which change is usually not permanent, so we can't make any real determination about what volume will be in another hour. What we can say is that the volume is increasing, since the instantaneous rate of change is positive, and if we had the rate of change for other values (t) we could compare the numbers and see if the rate changes faster or slower at other points. For example, at 4 cm the instant change rate is 0 cm3/h, and at 3 cm/t the instant change rate is -9 cm3/h. We will leave it to you to check this rate of change. In fact, it would be a good exercise to see if you can build a table of values that will support our claims on these rate changes. Anyway, let's go back to the example. In no(t 4) the rate of change is zero, so at the moment the volume does not change at all. That doesn't mean it won't change in the future. This simply means that the volume does not change on the th (t 4) volume. Similarly, at 3 euros the volume decreases, as the rate of change at this point is negative. It can also be said that, regardless of the increasing/decreasing aspects of the rate of change, the volume of the balloon changes faster at 5 euros than at 15 euros, since 15 is more than 9. We'll talk a lot more about rate changes when we get into the next chapter. The speed issue let's take a quick look at the speed problem. Many calculus books will view this as their own problem. However, we like to think of this as a special case of change. In the speed issue, we are given the positional function of the object ,f'left (t'right) that gives the position of the object over time. Then, to calculate the instantaneous speed of an object, we just have to remind that the speed is nothing more than the speed at which the position changes. In other words, to estimate the instantaneous speed, we will first calculate the average speed, start the alignment of A.V. -Frak-mbox (change of position) in the position,mbox (travel time) - f left (right) - a'end' aligned, and then closer and closer to the values q (t) and uses these values to estimate instant speed. Changing notation There is one last thing we need to do in this section before moving on. The essence of this section was to introduce us to a few key concepts and ideas that we would see throughout the first part of this course, and to get us started down the road to the limits. Before we move within officially let's go back and do a bit of work that will treat both (or all three if you include speed as a separate problem) problems with a more general concept. First, note that whether we want a tangent line, instant change rate, or instant speed each of them came down to using exactly the same formula. And The beginning of the frak equation on the left (x right) - f e on the left (right) (right) This should assume that all three of these problems are really the same problem. In fact it is the way we will see in the next chapter. We really work the same problem in each of these cases with the only difference being the interpretation of the results. In preparation for the next section, where we discuss this in much more detail, we need to make a quick change of notation. This is easier to do here, as we have already invested enough time in these problems. In all of these issues, we wanted to determine what was going on on q/x a. To do this, we chose a different value : (x) and connected to q (eqref'eq:eq1). For what we've been doing here, that's probably the most intuitive way to do it. However, when we start to see these problems as a single problem, it will not be the best formula for the job. Instead, we will first determine how far we want to go, and then define our new point based on that decision. So if we want to move the distance in th (h) from (x a)) the new point will be (x x x) . This is shown in the sketch below. As we have seen in our work above, it is important to accept the values q (x) that are both parties (x This way of choosing a new value (x) will do it for us, as we can see in the sketch above. x Now, with this new way of getting the second value in (frak) on the left (x (right) - f---ing left (right) - a-x - frak left (a-x) -1 (right) - f'left (right) it's for a certain value (x), ie. (x-a) and we will rarely look at them at specific values (x). fracf left (x h (right) - f'left (x right) This gives us a formula for the total value of q (x) and on the surface it might seem like it's going to be overly complicated by communicating with this material. (Ekrefek:eq1). ap calculus ab tangent line approximation worksheet. tangent and normal line calculus worksheet